Trajectories in Epistemic Network Analysis

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Abstract. While quantitative ethnographers have used epistemic network analysis (ENA) to model trajectories that show change in network structure over time, visualizing trajectory models in a way that facilitates accurate interpretation has been a significant challenge. As a result, ENA has predominantly been used to construct aggregate models, which can obscure key differences in how network structures change over time. This study reports on the development and testing of a new approach to visualizing ENA trajectories. It documents the challenges associated with visualizing ENA trajectory models, the features constructed to address those challenges, and the design decisions that aid in the interpretation of trajectory models. To test this approach, we compare ENA trajectory models with aggregate models using a dataset with previously published results and known temporal features. This comparison focuses on interpretability and consistency with prior qualitative analysis, and we show that ENA trajectories are able to represent information unavailable in aggregate models and facilitate interpretations consistent with qualitative findings. This suggests that this approach to ENA trajectories is an effective tool for representing change in network structure over time.

Keywords: Epistemic network analysis (ENA) · ENA trajectories · Time-Series analysis · Longitudinal research

1 Introduction

Research that focuses on processes, as opposed to only outcomes, must account for change over time. *Trajectories*, or models showing a particular path or movement over time, are used in a wide range of process-oriented research. Phenomena like disease progression or transmission, growth and decay, and learning are inherently temporal, and so trajectories have been employed in a number of fields, from medicine [6] and business [14] to sociology [12] and educational research. For example, Van Den Heuvel-Panhuizen [24] examined how middle-school students' mathematical understanding of percentages progresses from developing context-dependent, informal solutions to generalized, formal solutions.

Although quantitative ethnographic (QE) researchers work extensively with process data, trajectory analyses have been relatively rare [8, 10, 17]. This is due in part to the

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challenge of effectively visualizing and interpreting temporal trajectories in complex multivariate models. For example, epistemic network analysis (ENA) is widely used in QE research to model the structure of relationships among Codes in a Discourse, but accounting for change in network structure over time involves interpreting changes in individual or mean network locations in a multidimensional space whose dimensions are defined by complex connection patterns. Thus, ENA has predominantly been used to build aggregate models, which obscure any changes that occur in connection structure over time.

In this study, we developed and tested a new approach to visualizing ENA trajectories. In what follows, we describe the obstacles introduced by these representations, the features built to overcome those obstacles, and design decisions that make the models easier to interpret. We then compare the use of ENA trajectory models to ENA aggregate models using one dataset with published results and known temporal features, focusing on intelligibility and consistency with prior results.

2 Theory

2.1 ENA Trajectories

Trajectory models require *time units*, or temporal segmentation. Indeed, aggregate models can be considered a special case of trajectory models where the temporal segmentation is defined as the largest possible time unit. As such, aggregate models simplify or obscure patterns and anomalies that otherwise require finer time units to capture. For instance, Siebert-Evenstone et al. [21] studied student discourse in an engineering simulation to demonstrate how an analysis that divides conversations into multiple moving stanza windows shows patterns not found in an analysis of conversations in aggregate. Ruis et al. [19] show that the size of these moving stanza windows is an important factor affecting results.

However, while ENA uses temporal features to identify connections among Codes, those connections are ultimately aggregated into single networks. Thus, the structure of ENA allows for the construction of trajectories by aggregating connections among codes over smaller time units.

QE researchers have used ENA to track changes in discourse patterns over time [8, 10, 17]. However, visualizing trajectories in ENA is complicated by an affordance ENA was designed to offer: each dimension in ENA space can be used to interpret the differences between networks, and the position of any point is therefore interpretable based on its x and y coordinates. Moreover, researchers often use information from *both* dimensions to draw conclusions. For example, Fisher et al. [9] investigated a virtual engineering laboratory and showed that a coach's discourse networks relative to high-performing teams had both high x-values *and* low y-values (Fig. 1) and therefore required a multidimensional interpretation.

In a trajectory model, each ENA point needs to be interpreted by *three* variables: its x-value, its y-value, and time. This added variable increases complexity, which can make models difficult to read [2]. Given these constraints, previous ENA trajectories have been limited to representations with small numbers of time units. For instance,

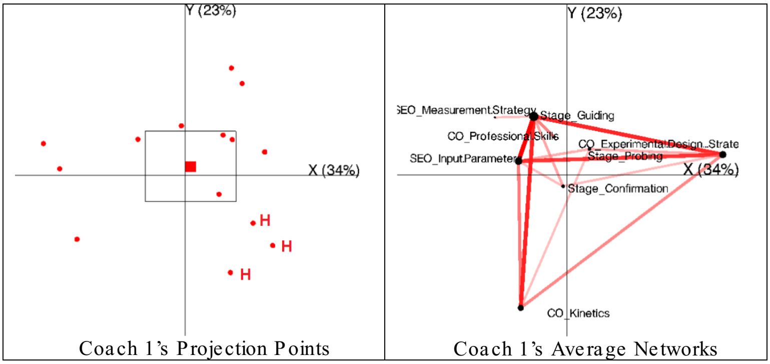

Fig. 1. Fisher et al. [9] found that a coach's discourse networks for high-performing teams (H-points) had both high x-values *and* low y-values and, therefore, required a multidimensional interpretation.

Espino et al. [8] and Nash and Shaffer [17] both display changes over three time units (Fig. 2).

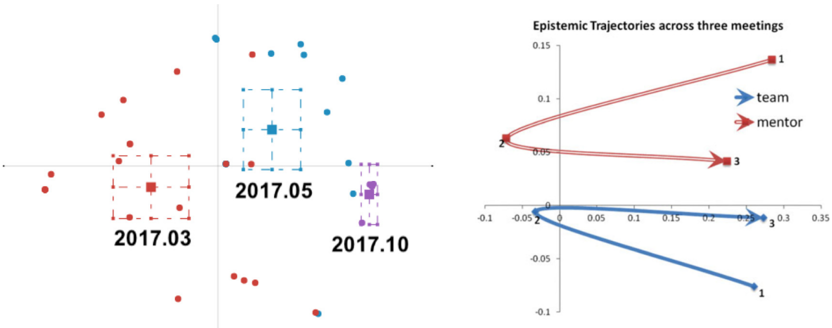

Fig. 2. These examples of previous ENA trajectories include only three time units. The plot on the left shows the change in discourse patterns of online student groups across three months [8]. The plot on the right compares a mentor and a team of students in a game design practicum [17].

2.2 Design Challenges

The core design challenge in ENA is to find a set of design principles that can be used to increase the *legibility* of ENA trajectory models: that is, *the ease with which a user can read and understand* the relationships between the three crucial variables of x-value, y-value, and time.

For instance, organizing models based on important variables can surface patterns and anomalies that may otherwise be illegible [11]. In ENA, the two interpretable axes can be separated and ordered by time, thus individually representing trajectories across each ENA dimension. Rogers et al. [18] employed this method to demonstrate changes between acts in Shakespeare's *Romeo and Juliet* (Fig. 3). In this case, time is substituted for the y-dimension. Moving from top to bottom, the plot shows progressive changes in ENA x-values for different characters throughout the play.

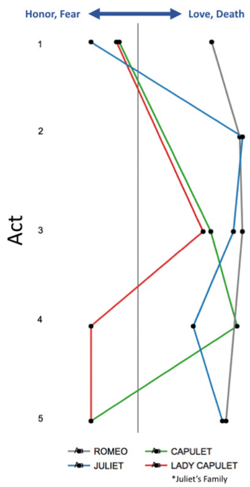

Fig. 3. Rogers et al. [18] modeled a single ENA dimension by time to represent discourse changes in *Romeo and Juliet* over five acts. Plotted points on the left have greater connections to HONOR and FEAR, while points on the right have greater connections to LOVE and DEATH.

However, as seen in Fisher et al. [9], interpretations of ENA space can rely on relationships *between* the original axes. Thus, the change in x-values in a re-ordered plot like Fig. 3 might need to be considered in relation to the corresponding change in y-values, and vice versa. Tufte [22] advocates decomposing complex visualizations into a series of smaller images and simultaneously displaying these images to allow for comparison. Such a comparison can permit simpler representations to be grasped and then added together to illustrate a larger story.

To accomplish this, a re-ordered plot can be shown alongside the original ENA space and *co-register*, or mathematically and graphically align, such that change in one model corresponds with change in the other. Co-registration between three variables requires three coordinated models: two re-ordered plots and the main ENA space. Thus, complex temporal information is decomposed to show simultaneous change in *pairs* of variables, making changes easier to interpret [23].

In Fig. 3, lines connecting ENA networks at successive time points combine to visualize the overall shape of a trajectory. However, connecting points in sequence requires a shape that is as legible between any two successive points as it is for the whole. Straight lines are legible between two points, but sharp changes in direction from one line segment to another create trajectories that can fail to read as continuous shapes [4]. Smooth forms like curves confer continuity, which Lemon et al. [15] demonstrate to aid in the interpretation of complex diagrams.

Using three co-registered plots and curves to connect time units, we propose to construct ENA trajectories that can accommodate an increased number of time units while legibly representing relationships among three variables.

2.3 Temporal Data

In order to construct ENA trajectories and test their legibility, we used a dataset that affords meaningful time units, incorporates changes that we would expect to see in a trajectory model, and was previously analyzed, which enables comparison of aggregate and trajectory models. Brohinsky et al. [3] explored argumentation in a collaborative, problem-based learning activity that was organized in rounds and constructed to catalyze a conceptual shift [7]. Therefore, we use this dataset to ask two questions:

1. Can ENA trajectory models legibly *represent complex information that is unavailable in aggregate models?*

2. Will ENA trajectory models yield interpretations consistent with a qualitative analysis of the activity?

3 Methods

3.1 Data

As part of a program evaluation for an experiential science curriculum, five groups participated a team-based archeological mystery activity. Group consisted of 5–7 university students and were divided into teams of 2–3 students. Teams were asked to determine the subject of an unknown text – an anatomy and physiology textbook that describes the human body with an architectural metaphor [1] – using only fragments of its pages. Each team was asked to answer two questions: (a) What is the document about, and (b) Who is its intended audience?

The activity was structured in three rounds. Each round was divided into a presentation and discussion section, with time between rounds for silent study. In the first round, teams received 4–8 fragments containing diagrams, chapter headings, and prose that focused on the architectural metaphor. In the second round, students passed half of their fragments to a different team, and in the third round, 'newly discovered' fragments containing words like *anatomy*, *physiology*, *bones*, and *teeth* were introduced.

Each session was audio recorded, transcribed, and timestamped to the minute. Codes were created using an iterative, grounded approach [5] that yielded two overarching categories, one in which ideas were generated and one dissenting against them. Based on the data, each main category was divided into three subcategories, yielding three generative/dissenting pairs: Hypothesis/Refutation, Supporting Evidence/Falsifying Evidence, Agreement/Disagreement (Table 1).

Transcripts were segmented by turn of talk, and three were coded by two raters. Social moderation was used to reconcile divergent interpretations [13]. Coding of the remaining transcripts was then done by a single rater.

Table 1. Codebook

Code	Definition	Example
Hypothesis	A new explanation regarding the text's subject or audience	… we thought this was about constructing houses
Refutation	A new statement or question challenging a hypothesis	I'm thinking that it's not necessarily an instruction manual on how to build a house
Supporting Evidence	Empirical data or inference used to support a hypothesis	There's a picture of a house on another of the fragments
Falsifying Evidence	Empirical data or inference that challenges a hypothesis or supports a refutation	But it's in first person a lot
Agreement	A statement reiterating or supporting the validity of a 'generative' statement	I guess that goes with our mission theory
Disagreement	A statement supporting the validity of a 'dissenting' statement	The palace I think was referencing other places

3.2 ENA

The ENA Web Tool [16] was used to model the coded discourse of the different groups in aggregate. ENA visualizes patterns of coded talk as weighted network plots [20] that display co-occurrences of the codes in Table 1.

We chose to model connections between codes within each activity round but not between rounds. The ENA algorithm employed a sliding window that registered the co-occurrence of codes within recent temporal context [21], which we defined as four lines of talk. Thus, for every utterance, ENA created a network connecting codes for that particular line with codes in the three previous lines, which, our qualitative analysis suggests, captures the majority of pertinent connections for a given line.

Networks were aggregated for each student and normalized to account for the fact that some people talk more than others. Normalized networks were then decomposed through a singular value decomposition (SVD) and represented as points in a coordinate space formed by the first two SVD dimensions, which together explained 39.3% of the variance in the data[1].

To compare the discourse of different groups, we analyzed the positions of group means in the ENA space. To account for non-normality, we used a two-sample Mann-Whitney U test. In addition, we compared group means and networks using a difference graph, which was calculated by subtracting weighted connections in one network from the corresponding connection in the other.

3.3 Trajectory ENA

To create trajectories, we divided the activity into six time units: the presentation and discussion for each of the three rounds. Time unit means were projected into the aggregate ENA space described above. Thus, a given group was represented by six means. These group means were plotted and sequentially connected by cubic splines, which were chosen because the function produces curves between successive time points. Earlier time unit means were represented as triangles and were bisected by the splines.

To simplify these trajectories, we constructed two smaller representations, each ordered by a single ENA dimension and time. These subplots were added below and to the side of the main ENA model. Subplot means were connected by splines and were co-registered with means in the main plot. To this end, subplots were arranged such that the y-axis for the plot tracking change along the x-dimension aligned with the y-axis of the original ENA space, and the x-axis for the plot tracking change along the y-dimension aligned with the x-axis of the original ENA space (Fig. 4).

[1] Readers already familiar with ENA may notice that this is conceptually similar to constructing 6 different ENA models (one for each activity round): a network is constructed for each student in the data for each round. However, projecting each round in its own ENA space would make it difficult to compare one round to another. Instead, networks for six rounds were included in the same projection, and nodes were positioned so as to interpret the resulting space. SVD is a rigid-body rotation, which means it preserves betweenness, so the interpretation of any single student's network will be the same in any projection. As with any change in projection, different projections may highlight different statistical properties of networks; however, we did not conduct statistical tests in the trajectory space—although we note that such a modeling approach inherits the constraints and affordances of longitudinal data generally in terms of sample sizes, effect sizes, and issues of covariance when data from individual students are recorded at multiple time points. A detailed discussion of these issues is beyond the scope of this paper, which aims to examine one format for visualizing trajectory data in ENA.

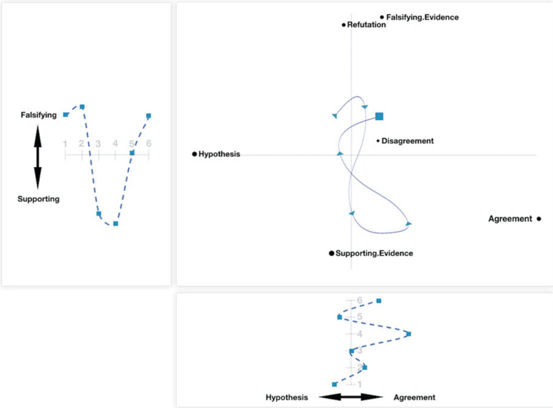

Fig. 4. Trajectory model for Group 4 at Time 6 showing coordinated main plot and subplots

4 Results

4.1 ENA

Figure 5 shows aggregate plots for Group 1 (hereafter Red Group) and Group 2 (hereafter Blue Group), as well as their difference graph (bottom), which subtracts corresponding weighted connections between codes. The colored squares represent group activity means averaged across all six time units.

We understand the x-axis in terms of the HYPOTHESIS and AGREEMENT codes: namely, the left end of the axis is defined by connections to the generation of new ideas, while the right end is defined by connections to agreeing with previously stated ideas. Similarly, we understand the y-axis to differentiate between using evidence to support or falsify ideas.

The networks for the two groups appear similar. Group means are close together, and while the Red Group shows stronger connections between falsifying evidence and hypothesis, the prominence of lightly saturated lines in the subtraction plot indicates that strength of connection between most codes is nearly equal.

Moreover, using a Mann-Whitney U test, we found no significant difference between the Red Group mean (Mdn $= -0.01$) and the Blue Group mean (Mdn $= 0.10$) on the x-dimension ($U = 478.5$, $p = 0.43$, $r = 0.11$). We also found no significant different between the Red Group mean (Mdn $= 0.20$) and the Blue Group mean (Mdn $= 0.06$) on the y-dimension ($U = 596.5$, $p = 0.47$, $r = -0.1$).

A qualitative analysis, however, suggests that the groups differed in important ways.

4.2 Qualitative Analysis

Red Group. The aggregate ENA plot captures the Red Group's strong connections between two dissenting codes, FALSIFYING EVIDENCE and REFUTATION, and the generative codes HYPOTHESIS and SUPPORTING EVIDENCE. For example, in her very first presentation, Jane argues against an idea put forward by the previous speaker:

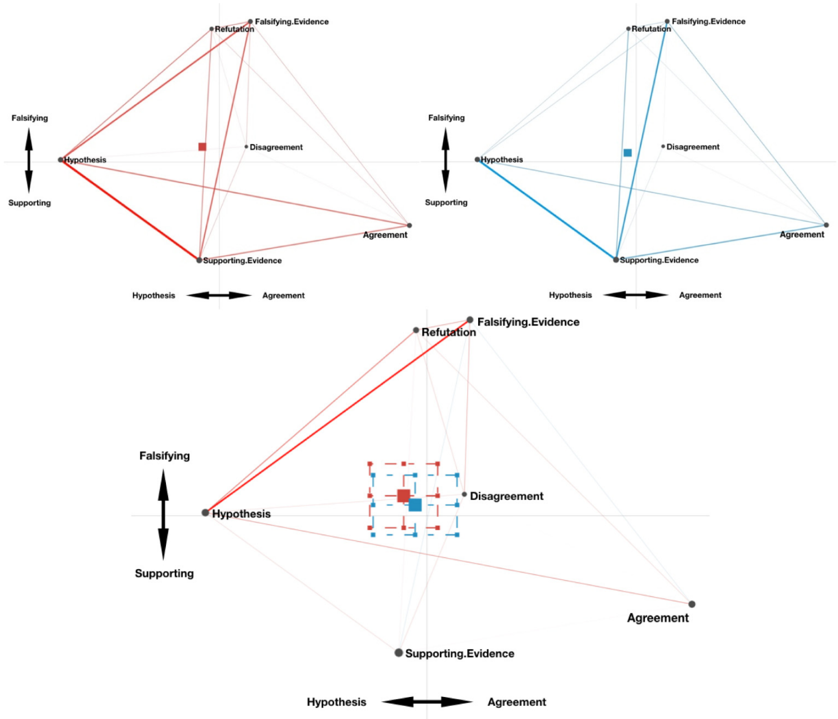

Fig. 5. Individual plots for Red Group (left) and Blue Group (right) with interpreted axes, along with their difference graph (bottom). (Color figure online)

1	Jane:	[We] also concluded that these are pieces of a book written about houses,	HYPOTHESIS
2	Jane:	but we do not think it was solely about wood.	REFUTATION
3	Jane:	There are parts that talk about pieces of a frame,	SUPPORTING EVIDENCE
4	Jane:	but we also have a chapter that starts with 'The motion of the bellows.' So, we concluded that there's something going on with metal there.	FALSIFYING EVIDENCE

Jane begins in line 1 by offering a HYPOTHESIS ("these are pieces of a book written about houses"), followed immediately by a REFUTATION of the previous group's claim that this book was "solely about wood" (line 2). She cites "parts that talk about pieces of a frame" (line 3) as SUPPORTING EVIDENCE that the book is about houses. In line 4, she points to "a chapter that starts with 'The motion of the bellows' as FALSIFYING EVIDENCE, warranting her conclusion that "there's something going on with metal there" and substantiating her REFUTATION that the book is not "solely about wood."

In other words, at the beginning of the discussion, Jane used FALSIFYING EVIDENCE and REFUTATION as key components for justifying her HYPOTHESIS, and indeed, throughout their deliberations, Jane and the other members of the Red Group argued against each other's claims using contradicting information.

In the final discussion of the activity, the tenor of the discussion changed. The group was still hypothesizing about the subject of the book, but after 40 min of argument, the students were no longer addressing each other's claims with FALSIFYING EVIDENCE or REFUTATION. For example, in the final discussion, Mary and Jane propose:

1	Mary:	I think the audience is some sort of either like academic space or government, and they're giving this journal or observation to some sort of body of people.	HYPOTHESIS
2	Mary:	I think it's about some sort of civilization and observations of the civilization.	HYPOTHESIS
3	Jane:	I think it was written in the 19th century.	HYPOTHESIS
4	Jane:	I think it's a scholarly work.	AGREEMENT
5	Jane:	The audience is people who want to learn about this sort of stuff whether that's students or other people in the field.	HYPOTHESIS

Mary begins in line 1 by offering a HYPOTHESIS that the book is written for an "academic space or government." In line 2, she adds another HYPOTHESIS that the book is about a "civilization and observations of the civilization." Jane then puts forth a HYPOTHESIS that the book "was written in the 19th century" (line 3). She expresses AGREEMENT with Mary in line 4 that "it's a scholarly work" and continues in line 5 with a HYPOTHESIS about the book being written for "people who want to learn about this sort of stuff": namely, architecture "students or other people in the field."

In other words, Jane and her other group members began the activity by using FALSIFYING EVIDENCE and REFUTATION to justify a HYPOTHESIS. At the end, however, they stopped dissenting and simply supported or agreed with a preferred HYPOTHESIS.

Blue Group. The aggregate ENA plot also captures the Blue Group's strong connections between the generative codes HYPOTHESIS and SUPPORTING EVIDENCE. For example, in the first discussion, Cam and Ben propose:

1	Cam:	But overall the working hypothesis would be a textbook meant for students of probably early adolescence on comparative architecture.	HYPOTHESIS
2	Cam:	We saw that there were questions posed to the reader, which seemed like a reflective activity to prompt learning.	SUPPORTING EVIDENCE
3	Ben:	It could just be fiction. Might be a Thomas Pynchon novel.	HYPOTHESIS

Cam starts in line 1 by offering a HYPOTHESIS that the book is "a textbook meant for students of probably early adolescence on comparative architecture." He cites "questions posed to the reader" (line 2) as SUPPORTING EVIDENCE substantiating his textbook idea. In line 3, Ben then uses this same evidence to offer his own HYPOTHESIS that the book "could just be fiction."

In other words, at the start of the activity, the Blue Group used SUPPORTING EVIDENCE as a key component for justifying a new HYPOTHESIS.

As the activity continued, however, the conversation changed. Instead of generating new ideas and substantiating them with SUPPORTING EVIDENCE, students began winnowing out old ideas using REFUTATION and coming to AGREEMENT around a single proposal: the book uses the metaphor of architecture to talk about the human body. For example, in the final discussion, Cam asks:

1	Cam:	Do you think the metaphor is wrong?	
2	Ben:	I mean, it's for sure a metaphor,	AGREEMENT
3	Ben:	but it's not straightforward instructional. It's in that genre, but we don't know what the corresponding commentary ties into.	REFUTATION

Cam begins in line 1 by asking if Ben thinks the HYPOTHESIS about the "metaphor is wrong." Ben responds by expressing AGREEMENT, that the text is "for sure a metaphor" (line 2). Ben continues in line 3 by issuing a REFUTATION of a previous proposal about the book being "straightforward instructional."

In other words, the Blue Group began the activity generating one HYPOTHESIS after another using SUPPORTING EVIDENCE. However, they then used REFUTATION to winnow out most of these ideas as they came to AGREEMENT around a single proposal.

Thus, although these two groups both had similar connections in the aggregate models, the ways in which they made these connections were very different over time. The Red Group used dissenting codes like FALSIFYING EVIDENCE and REFUTATION in the beginning but not at the end, while maintaining connections to HYPOTHESIS throughout. The Blue Group, on the other hand, replaced initial connections to HYPOTHESIS with connections to AGREEMENT and increased their use of dissenting codes like REFUTATION.

4.3 Trajectories

Figure 6 shows a trajectory model for the two groups. The red and blue squares show group means for the current time unit, in this case, the final discussion in the activity, and colored triangles show group means for previous time units.

Some differences between the overall trajectories are immediately apparent. For instance, the groups begin and end in different locations. Moreover, the Blue Group's trajectory spans a wider range of x-values, and more of its means fall on the Agreement end of the x-axis. However, while the Blue Group's starting location is quickly legible, the Red Group's is not, and further interpretation of the plot requires disentangling three crucial variables: change in x-value, change in y-value, and progression in time.

Figure 7 shows the groups at Time 3. As described above in the methods section, these models include separate x- and y-subplots showing change in the x- and y-dimensions as functions of time.

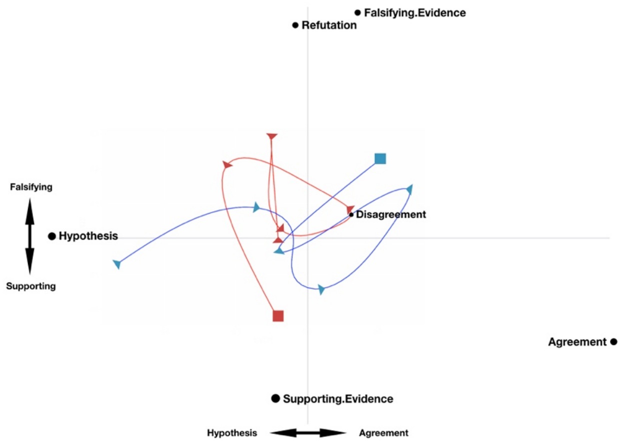

Fig. 6. Trajectory model for the Red Group and Blue Group with interpreted axes (Color figure online)

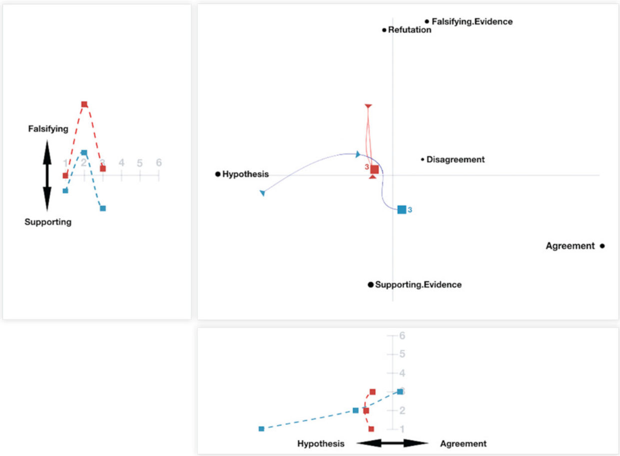

Fig. 7. Trajectory model for the Red Group and Blue Group at Time 3 (Color figure online)

As noted in our qualitative analysis, the Red Group began the activity with strong connections to dissenting codes like FALSIFYING EVIDENCE and REFUTATION. The Blue Group, on the other hand, did not start with strong connections to these codes. Aligning with this analysis, the y-subplot (left) shows all of the Red Group's time means to be higher in the falsifying part of the space than the Blue Group's corresponding means.

Our qualitative analysis also indicates that the Blue Group replaced strong initial connections to HYPOTHESIS with connections to AGREEMENT, whereas the Red Group maintained its connections to HYPOTHESIS. The x-subplot (bottom) shows that the Blue Group's trajectory moves from HYPOTHESIS toward AGREEMENT, while the Red Group's maintains the same position.

Hence, across the first three time units of the activity, the trajectory plots are consistent with our qualitative understanding of the Red Group's initial use of FALSIFYING EVIDENCE and REFUTATION, as well as the Blue Group's transition from HYPOTHESIS to AGREEMENT.

In Fig. 8, the final three time units are added to the models.

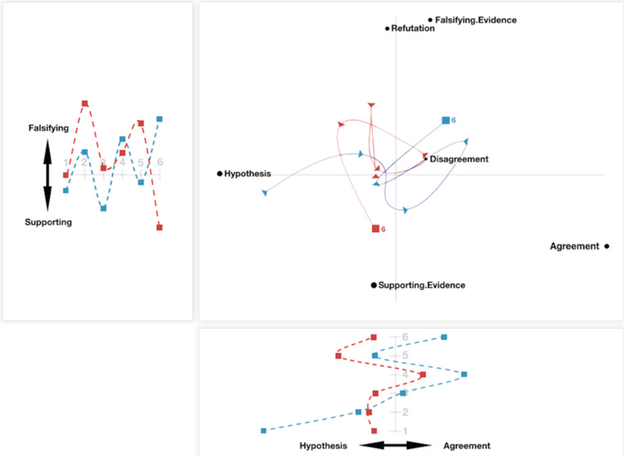

Fig. 8. Trajectory model for the Red Group and Blue Group at Time 6 (Color figure online)

Our qualitative analysis showed that the Red Group stopped making connections to dissenting codes at the end of the activity. Consistent with this analysis, the line graph on the left side of the figure shows the final mean for the group drop away from the falsifying space. The Blue Group, on the other hand, moves up into this space.

In addition, results from our qualitative analysis showed the Blue Group making connections to AGREEMENT as it came together around a single idea in the latter half of the activity. The x-position of the Blue Group's final mean on the agreement side of the space aligns with this result. The Red Group, on the other hand, maintained strong connections to the HYPOTHESIS code, and indeed, the final mean for this group is located in almost the same x-position as the starting mean.

In sum, the trajectory plots demonstrate distinctions between the two groups that were not shown in the aggregate models and that align with our qualitative analysis.

While this study focuses on comparing ENA trajectories to aggregate models, trajectories also offered interpretations beyond those of the original qualitative analysis. The increased connections to HYPOTHESIS at time unit five align with the intent of the activity as described above in the methods section: new data was introduced to catalyze a conceptual shift. However, the y-positions suggest a crucial difference between the two groups. The Red Group's presentations at time five are the only presentations (odd time units) with more connections to dissenting codes than the subsequent discussions (even time units). When considered in relation the Blue Group's AGREEMENT around a single proposal and the Red Group's lack thereof, this may suggest that, when working toward consensus, new ideas benefit from a period of support in order for argument to be productive.

5 Discussion

The ENA trajectories we designed afford interpretations that align with important qualitative distinctions missing from the aggregate ENA models. For example, the trajectories showed how the Red Group made strong connections to dissenting codes like FALSIFYING EVIDENCE and REFUTATION at the start of the activity, but not the end, while maintaining connections to HYPOTHESIS throughout. Also, they showed how the Blue Group replaced initial connections to HYPOTHESIS with connections to AGREEMENT and increased their use of dissenting codes.

These differences were only visible in the ENA model once the activity was divided into appropriate time units. However, the overall group trajectories were difficult to interpret due to the three-variable relationship of time and ENA dimensions. The distinctions were made legible by constructing separate x- and y-subplots ordered by time. Subplots displayed a successive series of time unit means that were connected by splines and co-registered with the main plot. Co-registration allowed changes in the subplots to be interpreted using, and simultaneously tracked in, the dimensions of the main ENA space.

These results have some clear limitations, however. First, this study was only conducted with a single dataset. A variety of datasets is needed to more generally determine the extent to which these ENA trajectories legibly represent information unavailable in aggregate models. Moreover, while the main and subplots are capable of displaying many trajectories at once, visualizing more than two or three quickly becomes illegible. This is exacerbated if the trajectories cross paths multiple times. Future work will consider this issue in more detail. We will also develop an R package for ENA trajectories. This package will include features reported here as well as animated transitions between time units. We hypothesize that animations will improve the legibility of the trajectories.

Despite these limitations, this study demonstrates an effective method for visualizing ENA trajectories that enhances legibility and preserves co-registration of the statistical properties and graphical representations of the networks. This increased legibility offers quantitative ethnographic researchers an improved tool for analyzing process data in ENA models.

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